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The Absorption of Heat Rays by Fog

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Measurements between two stations 0.4 km apart of the intensity of heat rays transmitted through fog showed that, although 7μ rays were slightly less absorbed by fog than visible light, their penetration into fog was too slight to be of practical interest. For a fog of visibility 0.6 km the distances to reduce the light to 10^{-2} were 910, 970, 980 and 1140 meters for wave-lengths 0.6, 1, 3 and 7μ, respectively.

MEASUREMENTS of the absorption of radiation by fog for various wave-lengths of light in the visible spectrum and in the infrared to about 3μ have shown that the infrared penetrated fog a little better than visible light.[1] In the present experiments the wave-length was extended to about 7μ and the conclusion is reached that, although these waves were slightly less absorbed by fog than the shorter ones, their penetration into fog was too slight to be of practical interest.

The experiments were carried out on the edge of the Potomac River, the receiver being 400 meters from the transmitter. The receiver was the same one used in the earlier experiment[1] and consisted of a thermopile with a fluorite window placed at the focus of a 20-inch metal mirror and connected to a sensitive galvanometer with a shunting resistance to keep the deflections on the scale. The transmitter was a boiler of blackened iron filled with boiling water at a pressure of 90 pounds absolute. The temperature of the water was 161°C. Assuming that the outside surface of the boiler was a black body at this temperature the spectral energy curve of the emitted radiation, after passing through the fluorite window of the thermopile, was calculated and is plotted in curve 1, Fig. 2. The curve is a maximum at 6.5μ and falls to a low value at 9μ because the fluorite became opaque at this wave-length. Tests with glass over the thermopile showed that the wave-lengths radiated by the boiler were longer than 3μ. We may refer to the boiler emission as "7μ radiation" although the rays were in a wide band from 5 to 9μ. The observations were not extensive enough to trace the variation, if any, of the atmospheric transmission with humidity.

The transmission of the atmosphere for the 7μ rays was observed during clear weather and on several days of atmospheric obscurity varying from haze to thick fog. The observations are plotted in the small circles of Fig. 1, the curve being their average. The ordinate is the transmission t for any epoque of fog and was obtained by dividing the galvanometer deflection for that epoque by the clear weather deflection. The abscissa is the visibility defined as the distance through the foggy atmosphere at which in daylight a dark object could barely be seen.

[1] Granath and Hulburt, Phys. Rev. **34**, 140 (1929).

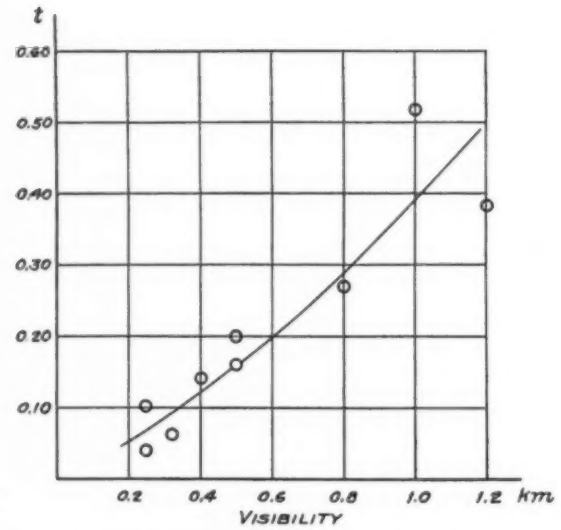

FIG. 1. Transmission t of 7μ rays through 0.4 km of foggy atmosphere as a function of the visibility.

From Fig. 1 it is seen that the amount of 7μ radiation reaching the thermopile decreased with the visibility. For visibilities below 0.4 km galvanometer deflections were produced, small to be sure, when the boiler was completely hidden by fog. This does not mean that a strong visible light at the distance of the boiler would have been obscured under these conditions. The situation is the same as that of an automobile with its head lights turned on approaching through fog in the daytime; one sees the lights at a greater distance than one sees the car.

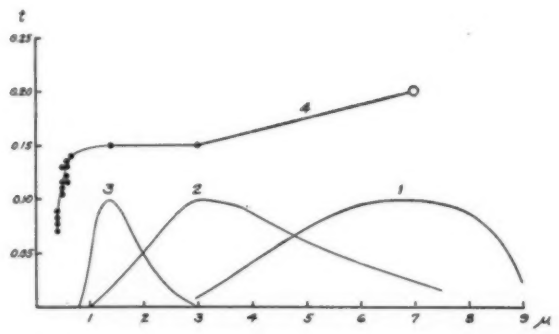

FIG. 2. Curves 1, 2 and 3, spectral energy curves of infrared transmitters, arbitrary scale of ordinates. Curve 4, transmission t through 0.4 km of foggy atmosphere of visibility 0.6 km as a function of wave-length.

From Fig. 1 $t=0.20$ for the visibility 0.6 km, this value of t is plotted as the small circle of curve 4, Fig. 2. The other dots of curve 4 are the values of t obtained in the former experiment,[1] curves 2 and 3 being the spectral energy curves of the infrared transmitters used at that time. From curve 4 the distances necessary to reduce the light to 10^{-2} of its original value were 710, 840, 910, 970, 980 and 1140 meters for wave-lengths 0.4, 0.5, 0.6, 1, 3 and 7μ, respectively. Therefore the longer infrared wave-lengths penetrated the fog slightly better than visible light, but not to such an extent as to suggest their utility for fog penetration. The conclusion is valid for fog thicker than fog of visibility 0.6 km, but cannot be safely extrapolated to the case of distances of several kilometers through thin haze.

Dielectric Recovery of a.c. Arcs in Turbulent Gases

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(Received July 24, 1933)

It is pointed out that arc extinction in modern high voltage alternating-current circuit interrupters is actually achieved by subjecting the confined arc to a turbulence-producing gas blast, thus accelerating the arc's dielectric recovery near current zero. Experiments to investigate the effects of gas velocity, gas composition, arc current magnitude, and constriction of the arc space upon the rate of recovery of dielectric strength by the arc near current zero are described. Results of these experiments show that most of the dielectric recovery occurring in a turbulent arc space within a hundred or so micro-seconds of current zero takes place before the actual moment of zero current. With a 600-ampere confined arc in carbon dioxide, it was found that the dielectric strength at current zero could be increased from 100 to 215 volts per centimeter by increasing the gas velocity from 35 to 110 meters per second. Dielectric recovery continued after current zero at about 25 volts per centimeter per hundred micro-seconds, and was nearly independent of gas velocity. A reduction in distance between confining walls from 1.9 cm to 0.635 cm resulted in a one-third increase of dielectric strength at current zero and a 100 percent increase in rate of recovery subsequent to current zero. Decreasing the arc current from 600 to 200 amperes resulted in a 120 percent increase in dielectric strength at current zero and an eight-fold increase in subsequent rate of recovery. In various gases, the comparative results for a 600-ampere arc are given in the table. These results are analyzed from the standpoint of recent theory.

Gas medium	Dielectric strength at current zero in volts per cm	Subsequent rate of dielectric recovery in volts/cm/100μ sec.
Air	70	13
Helium	115	26
Carbon Dioxide	145	25
Oxygen	170	7
Hydrogen	310	70

I. INTRODUCTION

IN all switches and circuit breakers used at high voltages final interruption depends on the extinction of a long a.c. arc. In high-power circuits the extinction always occurs at or near an instant of zero current. The opposing quantities which determine whether or not the arc goes out at a particular current zero are: (1) the voltage gradient across the arc space, and (2) the instantaneous dielectric strength of the arc space. The former can be measured or approximately calculated, and is not usually subject to control, but the latter can be influenced in a variety of ways.[1–5] All of the ways in present use, when analyzed, result in subjecting the arc to an intense, turbulence-producing blast of gas while it is confined to a restricted space. This paper deals with the recovery of dielectric strength during the current zero period by an experimental turbulent a.c. arc.

II. THEORY

Previous theory

In attempting to explain the observed rapid recovery of dielectric strength at current zero by the arc in modern gas-blast circuit interrupters, a number of different theories have been proposed.[4, 5] Of these, that proposed[3] and recently

* Work done at the Research Laboratories of the Westinghouse Electric and Manufacturing Company and submitted to the Graduate School of the University of Pittsburgh in partial fulfillment of the requirements for the degree of Master of Science. (California Institute of Technology 1933–34.)

[1] Van Sickle and Leeds, *Recent Developments in Arc Rupturing Devices*, Trans. A.I.E.E. **51**, 177 (1932).

[2] Baker and Wilcox, *The Use of Oil in Arc Rupture*, Trans. A.I.E.E. **49**, 431 (1930).

[3] Slepian and Denault, *The Expulsion Fuse*, Trans. A.I.E.E. **51**, 157 (1932).

[4] Prince and Skeats, *The Oil-Blast Circuit Breaker*, Trans. A.I.E.E. **50**, 506 (1931).

[5] Biermanns, E.T.Z. **50**, 1073–1079 and 1114–1119 (1929).

revised[6, 7] by Slepian seems best to withstand critical analysis in the light of the known facts. In this it was postulated that turbulence broke up the arc into a large number of separate parallel conducting filaments, which at current zero lost their conductivity very quickly because of diffusion of their ions into the surrounding gas. Results published by the writer[8] seemed to be in accord with this view.

Revised theory

Calculations showed, however, that in order to account for the large values of dielectric strength which a.c. arcs have been observed to possess very shortly after current zero on the basis of the diffusion from filaments occurring only after current zero, the initial diameter of the filaments would have to be absurdly small (of the order of 0.01 millimeter). Experiments with a suddenly-interrupted d.c. turbulent arc failed to discover such high recovery rates, however, those actually observed corresponding to calculated rates of diffusion from filaments of more reasonable dimensions (one millimeter).[7] These facts led to the conclusion that the picture of the extinction of an a.c. arc presented in previously developed theory had been over-simplified. It was no longer possible to think of the arc as a good conductor right up to current zero and then as a good insulator only a few micro-seconds later. The concept of dielectric strength of the arc under the dynamic conditions existing near a current zero was therefore re-defined by the author of the theory, not as that voltage at which considerable current begins to flow, but as that voltage at which the arc would just be stable, neither gaining nor losing conductivity. It was explained that this voltage lies on "a kind of static characteristic," whose voltage values for the same arc resistances (ratios of instantaneous arc voltages to currents) may be many times higher than those actually required by the arc in following its dynamic characteristic.

This may be seen more clearly by referring to

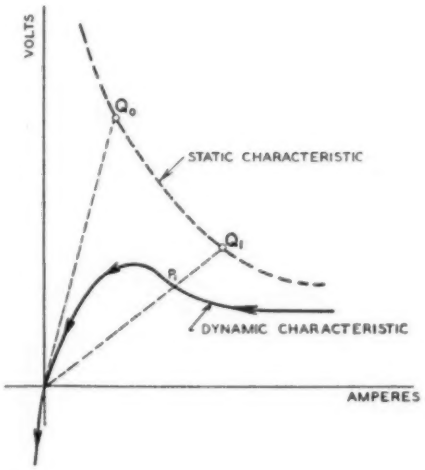

FIG. 1. Volt-ampere characteristic of arc.

Fig. 1, taken from Slepian's paper,[7] showing the descending portion of the arc's dynamic volt-ampere characteristic and the hypothetical static characteristic with which the dielectric strength of the arc is identified. If, as is conceivable, the breakdown voltage could be instantly applied to the arc space at some time before current zero, as at P_1, the voltage and current would immediately move along the line P_1Q_1 to Q_1 on the static characteristic. The line P_1Q_1 is, of course, the line through the origin corresponding to the instantaneous arc resistance at P_1. At actual current zero the resistance line will become tangent to the dynamic characteristic and its intercept on the static characteristic, Q_0, will be the dielectric strength of the arc space existing at current zero.

Dielectric strength of an arc space following a current zero has been thought of in the past as being equal to that potential gradient which would just fail to re-ignite the arc; if this value were exceeded, the arc would be maintained; if not, the arc would go out. Similarly from the new standpoint, if, shortly after current zero, the *gradient necessary to restore stability* to the arc is measurably exceeded, the conductivity of the arc space will stop decreasing and begin to increase. This will produce an increase in current which, because of the falling static characteristic, will further widen the margin between the applied voltage and that required for stability. This situation is clearly unstable, the processes aiding

[6] Slepian, *The Electric Arc in Circuit Interrupters*, J. Frank. Inst. **214**, 413 (1932).

[7] Slepian, *Extinction of an A.C. Arc in a Gas Blast*, Elektrotech. u. Maschinenbau **51**, 180 (1933). (In German.)

[8] Browne, *Extinction of A.C. Arcs in Turbulent Gases*, Trans. A.I.E.E. **51**, 185 (1932).

each other. Consequently, if the applied overvoltage lasts long enough, the arc space will quickly return to its original highly-conducting condition. If, however, the stability gradient is not reached by the applied voltage, the conductivity[9] of the arc space will continue to diminish until the current flow has entirely ceased, and the arc is then said to be extinguished.[10]

Dielectric recovery, according to new filament theory

As now pictured, the dielectric recovery of a turbulent a.c. arc at current zero is as follows: Just before current zero (perhaps a hundred micro-seconds or more) as the current is decreasing more and more rapidly, a point is reached where the existing filaments[11] are able to carry the comparatively small and now rapidly diminishing current without further energy input to them by the current. From this point on, the ionizing processes within the filaments practically cease, and they lose conductivity, and gain dielectric strength, at a rate which is determined only by the activity of the ever-present deionizing processes.

Conditions affecting deionization

Ions are believed to be lost from the filaments near current zero principally by diffusion. Recombination of ions should be of negligible importance *inside* the filaments, since experience and theory both indicate[12] that very little recombination is to be expected at temperatures above a thousand degrees or so. According to recent observers of atmospheric arcs, the initial temperature of the filaments will be 5000° or more, and so should not reach low enough values during the few hundred micro-seconds under consideration for internal recombination to become important. In the surrounding cooler gas, however, recombination undoubtedly does take place and so may have considerable indirect influence upon deionization of the filaments by increasing the ion density gradient across their boundaries, this gradient, of course, determining the rate of loss of ions by diffusion. The diffusion rate will be determined also by the "diffusion coefficient," which will depend upon the nature of the gas and the filament temperature, varying directly as the temperature and inversely as the square root of the mass of the positive ions. The importance of the initial diameter of the filaments has already been referred to. This is the quantity which will probably be most affected by the degree of turbulence of the gas in the arc space, since only filaments of small cross section will be able to exist in a highly turbulent gas, the larger filaments being torn apart by the turbulent motion. Increasing gas velocity, when the motion is turbulent, should diminish the average diameter of the existing arc filaments and so hasten their diffusion away at current zero, thus accelerating recovery of dielectric strength.

III. Rotating Arc Experiments

Theory

The general method for measuring the dielectric strength of a material is actually to apply various known voltage gradients to it and determine by observation the minimum value, on the average, which is required to break it down, or render it conducting. The application of this method to the a.c. arc space is complicated by the extremely rapid variations of the arc's dielectric strength with time, especially near current zero. Before current zero, no means have yet been devised for measuring the dielectric strength of the arc hypothetically defined above, since there is then already existing a definite arc voltage gradient which is generally different from this dielectric strength. After current zero, however, the voltage across the arc space can be controlled almost entirely by the connected circuit, since the "arc leakage" current is then

[9] The momentary conductivity after current zero of an arc being extinguished has actually been measured in studies with the cathode-ray oscillograph. (Van Sickle and Berkey, *Arc Extinction Phenomena in High Voltage Circuit Breakers*, Trans. A.I.E.E. **52**, 850 (1933).)

[10] The extinction or re-ignition of an a.c. arc was first described essentially as above by Slepian in his paper, *The Electric Arc in Circuit Interrupters*,[6] presented at the International Electric Congress, Paris, in 1932, but the term "dielectric strength" was not then used.

[11] Photographs of the arc in a compressed air switch (Biermanns, E.T.Z. **53**, 641 (1932)) seem to indicate that all of the current may be carried by a single filament near current zero.

[12] Slepian, *Flames from Electric Arcs*, Trans. A.I.E.E. **49**, 56 (1930).

small. Hence, the general method can then be applied, if the application of the measuring voltage can be accurately synchronized with a current zero and if means are available for observing whether or not this voltage causes breakdown of the arc space and consequent re-ignition of the arc after the current zero. In practice, this is fairly easy.

In these experiments the measuring voltage was furnished by the generator supplying the circuit in which the arc was drawn. In a simple a.c. circuit at current zero the voltage across the arc rises from the last value of actual arc voltage toward the maximum of the generated voltage following a volt-time locus, as shown in Fig. 2b,

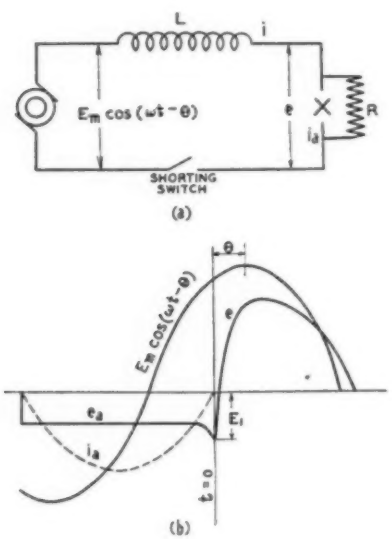

FIG. 2. Circuit diagram and volt-time relations.

determined by the power factor, the reactance, and the distributed capacitance of the circuit. In a purely reactive, zero power factor circuit the rate of rise to the peak generated voltage is limited only by the unavoidable distributed capacitance. In these experiments this factor was kept as small as possible and control of the rate of rise to peak voltage was obtained by means of resistance shunts, the resistance always being low enough to render the effect of distributed capacitance negligible. In calculating values of recovered voltage after current zero the circuit constants were assumed lumped, as in Fig. 2a, and current leakage through the arc space was neglected. Under these conditions the voltage would be determined by the relation, $e=Ri$, and the differential equation

$$Ldi/dt+Ri=E_m \cos(\omega t-\theta) \qquad (1)$$

where i = instantaneous current; R = shunting resistance; L = inductance of circuit; E_m = peak generated voltage; $\omega=2\pi f=377$ radians per second in these tests; θ = phase angle in radians by which current zero leads the peak generated voltage (see Fig. 2b); and t = time after current zero in seconds. Solving and substituting initial conditions, we have

$$e=\frac{E_m}{1+(\omega L/R)^2}\left(\cos(\omega t-\theta)+\frac{\omega L}{R}\sin(\omega t-\theta)-\left[\cos\theta-\frac{\omega L}{R}\sin\theta+\frac{E_1}{E_m}\left(1+\frac{\overline{\omega L}^2}{R^2}\right)\right]\epsilon^{-(Rt/L)}\right) \qquad (2)$$

which for convenience can be written

$$e=E_0[\cos(\omega t-\theta)+\beta\sin(\omega t-\theta)-\gamma\epsilon^{-\alpha t}] \qquad (3)$$

where

$$E_0=\sqrt{2}E/(1+\beta^2),\ \beta=\omega L/R=E/IR,\ \alpha=\omega/\beta,$$
$$\gamma=\cos\theta-\beta\sin\theta+E_1/E_0$$

and E = r.m.s. open-circuit voltage, I = r.m.s. short-circuit current in amperes, E_1 = arc voltage immediately preceding current zero.

Apparatus

The arcing structure (Fig. 3) has been described by the writer in a previous publication.[8] It was designed to obtain turbulence in the arc space similar to that produced by a gas blast without, however, having this turbulence accompanied by the cooling, diluting, and scavenging effects of a blast of fresh gas, to which gas-blast effectiveness has sometimes been wholly ascribed. To accomplish this, gas was led into the arc space only slowly and turbulence was obtained by driving the arc around the circular soapstone slot at high velocity by means of a radial magnetic field, produced by the coils shown. The arc was initially drawn in the slot, between the disk-shaped copper electrodes, by the burning of a fine copper fuse wire. The circuit also was the same as previously described.[8] In most of these experiments the reactance was

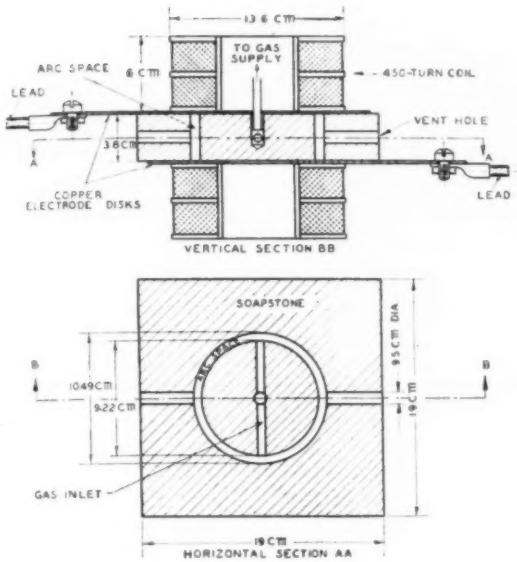

FIG. 3. Rotating arc structure.

adjusted so as to keep the short-circuit current at approximately 600 amperes. Only one half-cycle of arcing was used, this being initiated by a synchronous switch so as to keep θ (Fig. 2b) as small as possible.

Test procedure

At each of a series of open-circuit voltages the shunting resistance was adjusted by repeated trial to the maximum value at which the arc would usually go out at the end of the first half-cycle. For the frequent cases where the random variation of re-striking voltage from one test to the next was very large, the resistance was so adjusted that the arc was extinguished at the end of the first half-cycle in about one-half of the total number of trials. With these "critical" values of shunting resistance and other necessary data taken from oscillograms, "voltage rise" curves for each of the circuit voltages were calculated by means of Eq. (3) and plotted as in Figs. 4, 5, and 6. The envelope of such a system of curves is assumed to represent the dielectric strength of the arc space, or re-striking voltage of the arc, during the period of time explored by the test voltages.

Accuracy of method

The accuracy of results obtained in this way is limited by the accuracy with which the individual

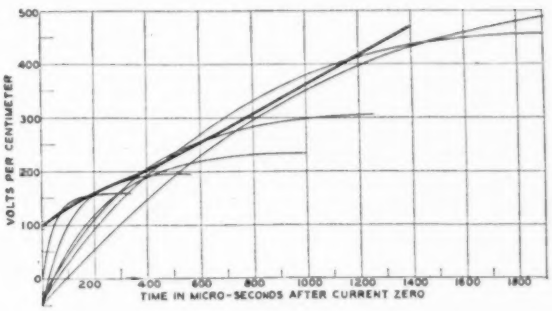

FIG. 4. Dielectric recovery by a 3.8 cm 600 ampere 60-cycle arc in CO_2 driven around a 1.9 cm circular soapstone slot at an average speed of 35 meters per second by a radial d.c. field of 163 gauss.

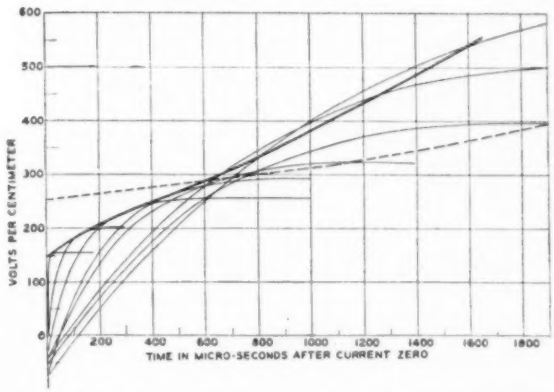

FIG. 5. Dielectric recovery by a 3.8 cm 600 ampere 60-cycle arc in CO_2 driven around a 1.9 cm circular soapstone slot at an average speed of 65 meters per second by a radial d.c. field of 340 gauss.

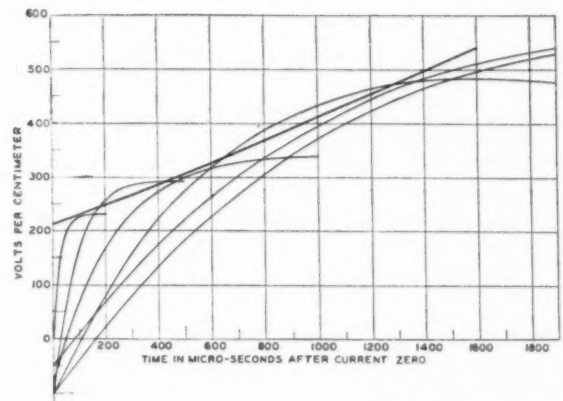

FIG. 6. Dielectric recovery by a 3.8 cm 600 ampere 60-cycle arc in CO_2 driven around a 1.9 cm circular soapstone slot at an average speed of 110 meters per second by a radial d.c. field of 700 gauss.

critical voltage rise curves could be determined. For the slower rates of rise the assumptions made in deriving Eq. (3) are undoubtedly justified, and measurements made from actual oscillograms of slowly rising transients check the calculations. For the transients reaching their peaks in the shorter times, however (perhaps 50 micro-seconds or less), the effects of distributed capacitance and of the conductivity of the arc space after current zero may become appreciable. Fundamental to this method of testing is the error resulting from the apparently random variation in arc and re-striking voltages from one trial to the next. It is believed that this is largely responsible for the scattering of the voltage rise curves on each side of the best possible smooth envelopes, in spite of the comparatively large number of tests made, Fig. 7 representing the results of 1560 trials.

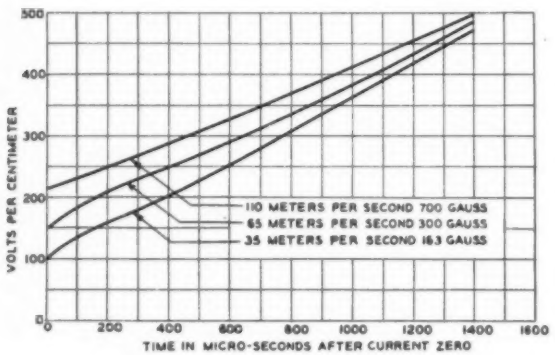

FIG. 7. Dielectric recovery by a 3.8 cm 600 ampere 60-cycle arc in CO_2 driven around a 1.9 cm circular soapstone slot at various speeds by means of a radial d.c. magnetic field.

It has been shown[13] that in undisturbed arcs of lengths such as that used in these experiments (3.8 cm) practically all of the dielectric strength at current zero is associated with the space charge regions at the electrodes. When the arc space is subjected to violent turbulence, however, the total voltage which it can withstand at current zero is multiplied many times,[8] and preliminary experiments showed this gain to be roughly proportional to the arc length, indicating that the added dielectric strength existed in the space between the electrodes. It is only this latter dielectric strength with which we are here concerned. Clearly, then, values calculated simply by dividing the total re-ignition voltage by the arc length will be in error due to concentration of voltage at the electrodes, the magnitude of the error depending on the arc length. This error cannot as yet be accurately corrected for, but since only comparative values are of interest in this work, its effect becomes of little importance if the arc length is held constant.

[13] Dow, Attwood and Timoshenko, *Probe Measurements and Potential Distribution in Copper A.C. Arcs*, Trans. A.I.E.E. **52**, 926 (1933).

IV. RESULTS

The first dielectric recovery envelope to be located by the method just described is shown in Fig. 5, along with the individual critical voltage rise curves. It is for a 600-ampere arc in carbon dioxide driven around the 1.9-centimeter slot at approximately 65 meters per second. The curve shown dotted is that published in an earlier paper,[8] and was obtained from a limited amount of data which were later found to be in error. This error, due to random variation, was hidden by the fortuitous occurrence of a smooth envelope curve. The dielectric strength thought to be recovered very near current zero was nearly 75 percent too high and the subsequent rate of recovery after current zero portrayed by the early curve was very much too low. The revised dielectric recovery curve is in much closer accord with Slepian's revised theory of dielectric recovery than the former curve, since extending it back into the region preceding current zero results in a picture very similar to Fig. 1 of his latest paper.[7] The curve of Fig. 5, so extended, would appear to reach arc voltage (about 30 volts per centimeter) two or three hundred micro-seconds before current zero. This time, by which the beginning of the dielectric recovery period apparently precedes actual current zero, is of the same order of magnitude as that estimated by Slepian.

In Figs. 4 and 6 are shown the results of further series of tests with the arc under the same conditions as in Fig. 5 except that in Fig. 4 the arc speed was reduced to 35 meters per second and in Fig. 6 it was raised to 110 meters per second.

Effect of arc speed

In Fig. 7 the envelopes from the foregoing figures have been replotted for easier comparison. In earlier studies[8] curves were obtained showing the effect of increasing speed of the arc on its interrupting ability in a fast (unshunted) circuit, but these curves show for the first time the effect of arc speed upon dielectric recovery by the arc during an extended period after current zero. Obviously, the effect is mainly upon the early stages of dielectric recovery close to current zero, and therefore is most effective in aiding the interrupting ability of the arc in a fast circuit. The effect is, in fact, entirely upon that portion of the recovery occurring before current zero, since the slopes of the curves after current zero are actually reduced by increasing the speed. The curves appear to approach each other at the longer times.

Effect of slot width

Keeping other conditions the same as in Fig. 5, further data were obtained with a slot only one-third as wide as that previously used. This was accomplished by increasing the diameter of the inner cylinder forming one wall of the slot to the value shown in Fig. 3. The envelope of these voltage rise curves is compared with the former one in Fig. 8. The field strength required to maintain the same speed in the narrower slot was increased about 40 percent. Comparison of these curves indicates that in the narrower slot the dielectric recovery was much more rapid, both before and after current zero, than in the wider slot.

Effect of arc current

Results of tests with a 200-ampere arc under the same conditions as was the 600-ampere arc of Fig. 5 are given in Fig. 9, and the 600-ampere envelope is repeated for comparison. The voltage scale is double that of the former figures. The increase in rate of dielectric recovery with the two-thirds reduction in arc current is noticeably much greater than the increase with the two-thirds reduction in slot width shown in Fig. 8. In this case the field strength was kept the same so that the speed of the 200-ampere arc was actually very much less than that of the 600-ampere arc: so much so, in fact, that it could not be reliably measured. These curves emphasize the fact that arc current is a very important factor in determining the rate of dielectric recovery near current zero, and therefore the voltage which can be interrupted by the arc. This should always be kept in mind when comparing and evaluating dielectric recovery data obtained in different experiments. Most of such data previously obtained in theoretical-experimental investigations of the arc, for instance, apply to arcs carrying currents at most of only a few tens of amperes, whereas the arcs in practical circuit interrupters which present the difficulties it is desired to learn how to overcome generally carry many hundreds or many thousands of amperes.

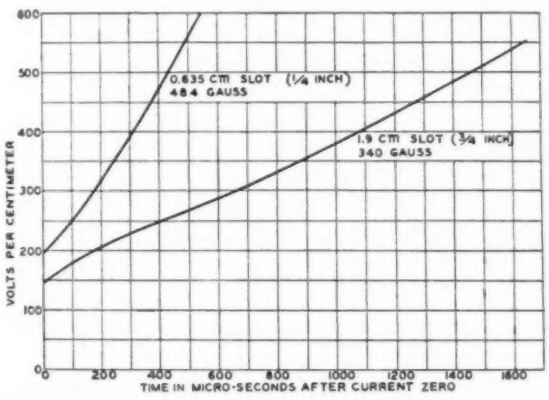

FIG. 8. Dielectric recovery by a 3.8 cm 600 ampere 60-cycle arc in CO_2 driven around circular soapstone slots of two different widths at an average speed of 65 meters per second by a radial d.c. field.

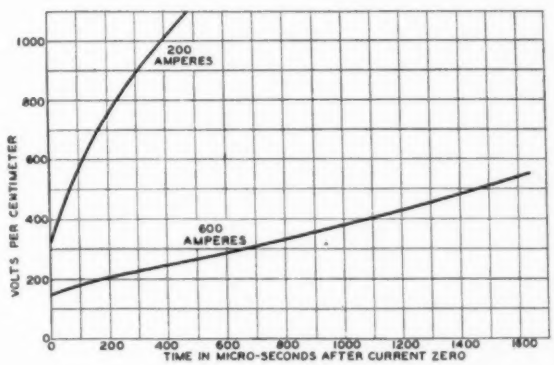

FIG. 9. Dielectric recovery by 3.8 cm 60-cycle 200 and 600 ampere arcs in CO_2 driven arount a 1.9 cm circular soapstone slot by a radial d.c. field of 340 gauss.

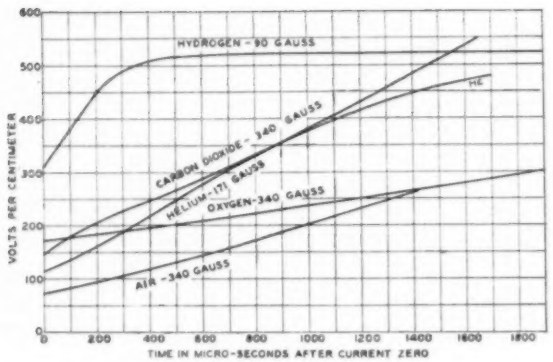

FIG. 10. Dielectric recovery by a 3.8 cm 600 ampere 60-cycle arc driven around a 1.9 cm circular soapstone slot at average speeds of 65 to 85 meters per second by a radial d.c. field in atmospheres of various gases.

Effect of gas medium

Fig. 10 shows the envelope curves from experiments with a 600-ampere arc under the conditions of Fig. 5 but in air, oxygen, helium and hydrogen, respectively, compared with that for carbon dioxide. In order to maintain approximately the same speed in the various gases, the magnetic field strength was varied. In air and in oxygen the field was the same as in carbon dioxide (340 gauss), but in the lighter gases, helium and hydrogen, it had to be reduced to 171 gauss and to 90 gauss, respectively. The measured average arc speeds varied from 65 to 85 meters per second. Because of the imperfect way in which the apparently best smooth curves could be fitted to the actual experimental voltage rise curves (not shown), the detailed shapes of the various envelopes may not be considered rigorously accurate, but certain general conclusions can be drawn from them.

In the first place, it is apparent that air is a poor medium for arc extinction, since dielectric recovery of the arc in air is relatively so slow, both before and after current zero. Previous results[8] indicate that pure nitrogen is a still poorer medium, and the much higher dielectric strength of the oxygen arc near current zero in Fig. 10 is in accord with this, at least as applied to fast circuits. At the longer times, however, the slower recovery after current zero of the oxygen arc space allows the dielectric recovery of the arc in air apparently to catch up with it at about 1400 micro-seconds. Helium is shown to be slightly inferior to carbon dioxide under the conditions of these experiments, especially in fast circuits. The curve for hydrogen is in agreement with past experience[3, 8] which has indicated that atmospheres of hydrogen, or hydrogen-containing gases, when turbulent, are far superior to other common gases as media for arc extinction.

Effect of slot material

Soapstone, of which the slots used in all of these experiments were constructed, is not a truly refractory material but has the property of giving off a certain amount of gas at temperatures approaching those of the arc. It was feared that the soapstone may, in this way, have influenced[3] the dielectric recovery of the arc. To test this, a new slot structure of the same dimensions as the original, with a 0.635-centimeter slot, was made and baked to render the material more nearly refractory. Results with this structure agreed within the usual experimental error with previous results; and so it was concluded that the composition of the insulating walls in these experiments with the arc in rapid motion was of no importance.

V. DISCUSSION

In Figs. 4, 5 and 6 it may be observed that the arc voltage immediately preceding current zero goes up with increasing arc speed. In fact, the average ratio of this value to the intercept of the envelopes on the zero-time axis remains practically constant at 40 percent, independent of arc speed. This suggests, as does Fig. 7, that the effect of turbulence in these experiments is entirely upon that portion of the dielectric recovery occurring before current zero. According to the filament theory, the effect consists of reduction in maximum size of the filaments, which speeds up diffusion of ions from them. Increasing turbulence must also bring about more effective cooling and deionization of the gas immediately surrounding the filaments, still further hastening their deionization. Since the accelerated deionization apparently does not continue after current zero, the filaments must have already been dispersed by the time the current has reached zero.

Effect of magnetic field

In addition to arc speed, the strength of the driving magnetic field increases progressively in Fig. 7 from one curve to the next, somewhat faster than the arc speed which it produces. In the earlier turbulent arc paper[8] it was concluded that this magnetic field had no direct influence on the extinction of the arc. Results of later experiments, as yet unpublished, discredit this conclusion, however. They show that, when turbulence is produced by another means such as a gas blast, an added magnetic field existing at current zero may considerably hasten dielectric recovery of the arc. This effect is not well understood at present. Calculations made in connection with the earlier paper[8] indicate that the effect of the magnetic field strengths used in these experiments upon the mobility and diffusion coefficients of the ions and electrons in the arc should be negligible. The observed effect may be due, however, to the force exerted by the field on the arc filaments just before current zero. This force, acting only on the conducting cores of the filaments, would tend to move the individual filaments through the gas immediately surrounding them in addition to causing observable bodily motion of the whole arc region, which alone has been heretofore considered. This would be equivalent to a transverse "draft" of gas through the arc past the filaments, further aiding deionization of the space between the filaments and so speeding diffusion of ions from them. Thus, the action of the magnetic field before current zero may help to simulate arc conditions existing in an actual gas-blast switch. This effect does not continue after current zero, of course.

Slot width

The higher interrupting ability of the arc in the narrower slot, as shown by Fig. 8, is in accord with past experience and approximate theory[14] but the interruption gradients in this case are much higher than for the undisturbed arcs previously investigated. Here, the greater turbulence resulting from the same arc speed in the more constricted slot is probably largely responsible for its superiority, although the increased slope of the envelope after current zero suggests that the greater diffusion of ions, and possibly also of heat, to the walls may also be important. Undoubtedly, the increase in magnetic field strength also aided dielectric recovery before current zero to some extent.

Arc current

The striking effect of arc current in Fig. 9 is difficult to understand on the basis of the filament theory unless it is assumed that only one filament exists near current zero. In this case, the slower rate of change of current near current zero in the smaller current arc would allow the arc resistance (Fig. 1) to reach relatively higher values as current zero is approached, since the dynamic arc characteristic tends to approach the static characteristic as the rate of change of current is diminished. In other words, at smaller effective currents, there is a smaller lag of conductivity of a single filament behind the instantaneous current, so that at a cyclic current zero the conductivity of the smaller current arc reaches very much smaller, or its dielectric strength very much greater values, than that of the higher current arc. Also, since the smaller current arc filament would be of smaller diameter and so would have a greater ratio of surface to volume, the rate of deionization by diffusion from it would be effectively greater at a given instant, and likewise its rate of cooling would be greater than for the higher current arc.

Gas medium

The faster dielectric recovery of the arc in oxygen as compared with the arc in air is believed to be due, at least partly, to the comparatively high rate of ion recombination in oxygen. In pure nitrogen, as in the noble gases, electrons do not attach themselves to molecules to form negative ions as they do very readily in oxygen;[15] and without this attachment, appreciable recombination does not occur. In air at ordinary temperatures, all of the electrons would become attached to oxygen molecules almost as quickly as in pure oxygen. At several thousand degrees, however, the rate of attachment will undoubtedly be very much smaller and the large proportion of nitrogen in the air may then have a considerable effect in slowing up electron attachment and therefore loss of ions by recombination. Since the

high diffusion gradient at the boundary of the atmospheric arc is due largely to recombination there, a considerable difference in rate of recombination due to difference in composition of the gas should result in a corresponding difference in the rate of deionization of an arc filament by diffusion. If, as already indicated by Fig. 7, the high diffusion gradients have disappeared by the time current zero is reached, this effect should not persist after current zero, which is in accordance with the curves of Fig. 10. The slower after-current-zero recovery of the oxygen arc may be the result simply of its lower initial maximum ion density at current zero, since loss of ions by diffusion is proportional to the first power and loss by recombination to the second power of the existing ion density.[14]

At the high temperature of the arc space, calculation from existing thermodynamic data shows that carbon dioxide should be very largely dissociated (99 percent at 5000°K) into oxygen and carbon monoxide. The presence of the latter, which has a relatively small attachment rate,[15] may have an effect in reducing recombination in the mixture similar to that which nitrogen is believed to have in air, thus explaining the fact that at current zero the dielectric strength of the arc in carbon dioxide was somewhat lower than that of the arc in pure oxygen. The faster dielectric recovery in carbon dioxide after current zero is difficult to understand.

In helium there is also no electron attachment and therefore no recombination, except that due to impurities, making the low dielectric strength of the helium arc at current zero appear to be reasonable. According to the supplier, practically the only impurity in the commercial helium used was nitrogen, amounting to one or two percent. This should not affect recombination, but would undoubtedly result in the current being carried almost entirely by nitrogen ions, since a sufficient number of nitrogen ions could easily be produced at gradients too low for appreciable ionization of the helium, whose ionizing potential is nearly 50 percent higher than that of nitrogen. The probable high concentration of metastable helium atoms[16] of energy sufficient to ionize nitrogen molecules by "collisions of the second kind" would also insure the existence of enough nitrogen ions to carry the current in the helium arc. The diffusion away of ions, then, should be little faster than for the arc in air, but cooling and diffusion of metastable atoms will be much faster in the light helium than in the heavier gases, oxygen, nitrogen and carbon monoxide. The dielectric recovery after current zero in helium was actually more rapid than in air and in oxygen, but was about the same as that in carbon dioxide.

The high rate of dielectric recovery of the arc in hydrogen was to be expected because of the high rate of diffusion of the very light hydrogen ions, which may be expected to exist. The chief impurity in the electrolytically produced hydrogen, 0.4 percent of oxygen, would probably be somewhat more intensely ionized than the hydrogen because of its lower ionizing potential (13 volts as compared with 16 volts), but in view of the small difference, the ions in the hydrogen arc space should be mainly hydrogen ions. Although some authorities[15] state that negative hydrogen ions are not formed, the coefficient of recombination in hydrogen is generally quoted[15] as having a value close to that for oxygen. The practical cessation of dielectric recovery in hydrogen apparently occurring at about 300 micro-seconds after current zero is as yet unexplained. It may possibly be due to the remaining behind of slow diffusing ions of impurities in the gas.

Conclusions

The arc conditions under which these results were obtained were intended to duplicate as far as possible the essential conditions existing in modern gas-blast circuit interrupters, while eliminating non-essential conditions and permitting the variables to be independently controlled. The chief condition eliminated, the cooling and scavenging action of an actual blast of fresh gas, may have had an appreciable effect on dielectric recovery, especially after current

[14] Slepian, *Extinction of a Long A.C. Arc*, Trans. A.I.E.E. **49**, 421 (1930).

[15] Compton and Langmuir, *Electrical Discharges in Gases*, Rev. Mod. Phys. **2**, 192 (April, 1930).

[16] Found and Langmuir, *Study of a Neon Discharge by Use of Collectors*, Phys. Rev. **39**, 237 (1932).

zero, but this effect is believed to be of secondary importance during the two or three hundred important micro-seconds near current zero. The magnetic field used to obtain turbulence had an accelerating effect on dielectric recovery before current zero which probably tended to compensate for the absence of the gas blast, but its action is not yet well understood. It is believed, however, that these dielectric recovery curves do at least approximately show the effects near current zero of independent variations within limited ranges in gas velocity (turbulence), proximity of enclosing walls, arc current, and gas medium upon the rate of dielectric recovery by an arc such as is utilized in gas-blast circuit breakers and fuses.

On the Theoretical Determination of Earth Resistance from Surface Potential Measurements

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A general solution for the potential in the steady flow of current from a single point electrode when the conductivity is an arbitrary continuous function of position is given in the form of an infinite series, by using the method of successive approximations, and conditions for the validity of the solution discussed. The converse problem of determining the conductivity when the surface potential is known is then shown to lead to an integral equation, which has no unique solution. The problem can be made determinate, however, by restricting the functional form of the conductivity—in particular by supposing that it is a function of depth only—or by supposing that the electrode is movable, and that the surface potential is known for all positions of the electrode on some curve at the surface. It is shown how the integral equations can be formally solved by the method of successive approximations. For the special case where the conductivity is a function of depth only, the first approximation is worked out in detail, and an approximate method is given for solving the resulting integral equation for the conductivity. This method is critically discussed and compared with the more exact method of Slichter and Langer. A numerical example is worked out where the conductivity is known in advance, namely a special case of the three-layer earth. It is shown that the present method gives a rough indication of the true behavior of the conductivity, but that the Slichter-Langer method and the "apparent resistivity" method both fail completely for this case. It is suggested that in certain cases the present method might give a better result than that of Slichter and Langer.

I. INTRODUCTION

THE problem of determining earth resistance when the potential due to the steady flow of current from a single point electrode is known at all points on the surface of the ground is of practical importance in geophysical prospecting. The usual method has been to calculate the surface potential on the assumption that the conductivity, which we shall denote throughout the paper by σ, varies according to some simple law, and then to compare the surface potential found in actual cases with that so calculated; if the two agree closely, it is supposed that σ does in fact vary in the manner assumed. This "trial and error" method is severely limited, due to the difficulties of calculating the surface potential when σ varies. Almost all calculations have been made on the assumption that σ is a function of depth only, and have usually been restricted to the case of an earth consisting of a certain number of layers within each of which σ is constant. Some of the most recent calculations of this type are those of King[1] and Muskat,[2] who have treated independently somewhat similar problems.

Recently, however, Slichter[3] and Langer[4] have developed a method for calculating σ from the surface potential which applies, with certain restrictions, to the case where σ is any function of depth only. In the same paper Slichter has also solved the converse problem of determining the surface potential when σ, being a function of depth only, varies continuously according to a number of special laws, and Professor King has informed the writer that he has also made calculations of this type in a forthcoming paper. Evidently much still remains to be done, however, from both the theoretical and practical standpoints.

The present paper attempts to give a general theory by showing how a formal solution of the problem of determining the potential when σ is assumed to be an arbitrarily assigned continuous function of position (and not necessarily a function of depth only) may be obtained by the

[1] L. V. King, Proc. Roy. Soc. **A139**, 237 (1933).

[2] M. Muskat, Physics **4**, 129 (1933).

[3] L. B. Slichter, Physics **4**, 307 (1933). Slichter also considers briefly the case of an anisotropic medium.

[4] R. E. Langer, Bull. Am. Math. Soc. **39**, 814 (1933).

method of successive approximations. The converse problem of determining σ from a knowledge of the surface potential is then shown to lead to an integral equation *which possesses no unique solution*. If, however, σ is assumed in advance to be a function of depth only (in which case the surface potential must, of course, be symmetrical with respect to the electrode), there exists a unique solution. This last conclusion is also reached by Slichter and Langer, and is doubtless implied in most of the theoretical work that has been done. In the general case, it is further shown that *σ is uniquely determined if the surface potential is known for all positions of the electrode on any curve at the surface.* It is perhaps worth noting that whereas in most problems of classical mathematical physics it can be seen intuitively whether there is a unique solution or not, under given conditions, even though it may be difficult to establish this mathematically, this is not so in the present case—at least so it seems to the writer.

The validity of the method used in calculating the potential when σ is assigned is established rigorously (apart from convergence of the series) if σ satisfies a certain condition. No attempt has been made, however, to establish rigorously the existence of solutions of the integral equations which occur in the converse problem, and which are of singular type; but it is reasonable to suppose that they possess unique solutions under proper conditions for the surface potential.

As regards the practical utility of the method here given, its application to the general case leads unfortunately to an integral equation which it has not been found possible to solve, but in the case where σ is a function of depth only an approximate method of solution is given. In the latter case, we merely obtain a certain approximation, whereas the Slichter-Langer method gives an exact solution where it is applicable. In many cases, however, the Slichter-Langer method is inapplicable, and it is suggested, on the basis of a numerical example which is worked out, that the present method might give a rough approximation even in such cases. Both methods are critically discussed more fully below.

2. General Solution of the Problem of Determining the Potential when σ is Assigned

We shall confine ourselves to the case where σ is a continuous nonvanishing function of position. Choosing rectangular axes with the point electrode as origin and the z-axis vertically downwards, the potential V must satisfy the equation div $(\sigma \nabla V) = 0$, or

$$\nabla^2 V = -\nabla f \cdot \nabla V, \qquad f = \log \sigma \tag{1}$$

subject to the boundary conditions:

(a) V single-valued and bounded for all bounded values of x, y, z, except at the origin where

$$V \to C/r, \qquad r^2 = x^2 + y^2 + z^2, \qquad C = I/2\pi\sigma_0,$$

I denoting the current in steady flow through the electrode, and σ_0 being the conductivity at the electrode ($\sigma_0 = \sigma(0, 0, 0)$).

(b) $\partial V/\partial z = 0$ when $z = 0$.

(c) Condition at infinity: if S is an infinite hemisphere center the origin,

$$-\int_S \sigma(\partial V/\partial n) dS = I.$$

For the normal case considered here where σ remains finite and positive at infinity, this means that V vanishes to order $1/r$ at infinity, and this may replace the above condition. If, however, $\sigma \to \infty$ or 0 as $r \to \infty$ this is not the case; if $\sigma \to \infty$ over all but an infinitesimal fraction of S, V vanishes to an order higher than $1/r$, while if $\sigma \to 0$ in a similar manner, V vanishes to an order less than $1/r$, or may tend to $-\infty$ (for instance, if there is a nonconducting layer at a finite depth, it is easily shown that V has a logarithmic singularity at infinity in the region where there is a flow of current). Either of these cases might be closely approached in practice, so that they must be reckoned with.

We proceed now to obtain a formal solution of (1) by the method of successive approximations, the "zero-approximation" being that of *constant* σ, and the term on the right-hand side of (1) being treated as a kind of perturbation. Assume then

$$V = V_0 + V_1 + V_2 + \cdots \tag{2}$$

where the successive approximations satisfy

$$\nabla^2 V_0 = 0 \tag{3}$$

$$\nabla^2 V_n = -\nabla f \cdot \nabla V_{n-1}, \qquad (n \geq 1). \tag{4}$$

(3) is the equation corresponding to constant σ, and has the elementary solution

$$V_0 = C/r \tag{5}$$

where C is a constant. We shall take the constant conductivity from which the zero-approximation is derived to be that at the electrode, so that C has the same value as before, namely $I/2\pi\sigma_0$.

(4) is Poisson's equation and has the well-known solution

$$V_n(\mathbf{r}) = \frac{1}{4\pi}\int \frac{\nabla' f(\mathbf{r}') \cdot \nabla' V_{n-1}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau' \tag{6}$$

where $\mathbf{r}$ denotes the vector (x, y, z), $\mathbf{r}'$ the vector (x', y', z') and $d\tau$ is an element of volume. The volume integral is taken over the half-space $z \geq 0$; in order to satisfy the boundary conditions, however, we define f for negative z by $f(x, y, -z) = f(x, y, z)$ and extend the integration over all space, which simply amounts to filling the half-space $z<0$ with conducting matter which is the image of that in the half-space $z>0$.

(6), combined with (5), defines V_1, V_2, $\cdots$ in succession, so that if the boundary conditions are satisfied and the series (2) converges we have the required solution. We proceed to show that the boundary conditions are satisfied if the function f satisfies a certain condition. Consider the first approximation, which is given by

$$V_1(\mathbf{r}) = \frac{1}{4\pi}\int \frac{\nabla' f(\mathbf{r}') \cdot \nabla' V_0(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\tau' = -\frac{C}{4\pi}\int \frac{\nabla' f(\mathbf{r}') \cdot \mathbf{r}'}{r'^3 |\mathbf{r}-\mathbf{r}'|} d\tau'.$$

Let $M(r)$ be the upper bound of $|\nabla f| = (1/\sigma)|\nabla \sigma|$ over the surface of the sphere $|\mathbf{r}| = r$. Then

$$|(\nabla' f(\mathbf{r}') \cdot \mathbf{r}')/r'^3| \leq M(r')/r'^2,$$

so that

$$|V_1| \leq (C/4\pi)\int (M(r')/r'^2) d\tau'/|\mathbf{r}-\mathbf{r}'|. \tag{7}$$

The integral in (7) is the potential at (x, y, z) of a volume distribution, spherically symmetrical about the origin, of density $M(r)/r^2$, so that, by a well-known result in potential theory

$$|V_1| \leq (C/r)\int_0^r M(r')dr' + C\int_r^\infty M(r')/r' dr'. \tag{8}$$

Hence a sufficient condition for the absolute convergence of the integral for V_1, for finite non-zero values of r, is that $\int^\infty M(r')dr'$ should converge. Further, if this condition is satisfied,

$$V_1 \sim (C/r)\int_0^\infty M(r')dr'$$

for large r. The only possible singularity of V_1 occurs at the origin. As $r \to 0$ the first integral in (8) has no singularity, while the second behaves like $CM(0)\int_r (dr'/r')$. Thus V_1 has in general a logarithmic singularity at $r=0$; if, however, $\nabla f = 0$ at $r=0$, V_1 remains finite there. Moreover, from (8) we certainly have $|V_1| \leq C'/r$ for all r, where C' is a constant, so that the argument above may be repeated for the higher approximations and the same results hold for them as for V_1.

These results, combined with the expression (5) for V_0, show that the boundary conditions (a) and (c) are satisfied by all the successive approximations V_i for V. Lastly, from Eqs. (5) and (6), and the definition of f for negative z, it is evident that the V_i are even functions of z, so that condition (b) is satisfied. Thus, *a sufficient condition for the validity of the solution given by (2), (5), and (6) (apart from convergence of the series) is that $\int^\infty M(r)dr$ shall converge.*[5] This condition is satisfied if σ tends to a constant non-zero value sufficiently rapidly as $r \to \infty$, for instance if there is a region of constant σ at a large distance from the electrode. From one point of view, this is not a serious restriction, for we are at liberty to impose any conditions on σ we please at infinity since, from the practical standpoint, only a finite portion of the medium is accessible.

As regards convergence of the series (2), it would be easy to obtain some sort of sufficient

[5] It can be seen that it is sufficient in this to consider $M(r)$, for large r, to be the upper bound of $|\nabla f|$ over *all but an infinitesimal fraction* of a sphere of radius r.

(though not necessary) condition, but this would be of little practical use. It is evident that it is convergent if the variations from constant conductivity (more precisely, the ratio of these variations to the mean conductivity) are sufficiently small, and that the smaller these variations are, the better the convergence will be.

Unfortunately, the case of most practical interest is where there are *large* variations, or even a discontinuity, in σ. A discontinuity might, in principle, be treated as a limiting case of a continuous variation, but evidently the method of successive approximations then ceases to be valid. Further, the cases where σ becomes infinite or zero must obviously be excluded. Thus the method is of restricted validity, and is ill-adapted to cases of practical importance; in spite of this, it is of theoretical interest as throwing light on the converse problem of determining σ from the surface potential, and, as we shall see, may be of practical use in some cases.

3. Determination of σ from Surface Potential

Let V_s denote the surface potential, so that $V_s(x, y) = V(x, y, 0)$. Using (6) to express the successive approximations in terms of $V_0 = C/r$, and then putting $z = 0$, we obtain from (2)

$$V_s(x, y) = \frac{C}{r_0} - \frac{C}{4\pi}\int \frac{\nabla' f(\mathbf{r}')\cdot\mathbf{r}'}{r'^3|\mathbf{r}_0-\mathbf{r}'|}d\tau' - \frac{C}{4\pi}\int\int \frac{(\nabla' f(\mathbf{r}')\cdot\mathbf{r}')(\nabla'' f(\mathbf{r}'')\cdot\mathbf{r}'-\mathbf{r}'')}{r'^3|\mathbf{r}_0-\mathbf{r}''|\,|\mathbf{r}'-\mathbf{r}''|^3}d\tau' d\tau'' - \cdots \qquad (9)$$

where $\mathbf{r}_0 = (x, y, 0)$, $r_0 = |\mathbf{r}_0|$.

If we now regard V_s and C as known, this is an integro-differential equation of a very complicated type for the determination of the unknown function f. In order to see if it possesses a solution, it is sufficient to confine ourselves to the first approximation, so that we have an integral equation of the form

$$\phi(x, y) = \int \nabla' f(\mathbf{r}')\cdot\mathbf{K}(x, y; \mathbf{r}')d\tau', \qquad (10)$$

where ϕ, $\mathbf{K}$ are known functions. It is seen at once that (10) possesses no unique solution for f, since the left-hand side is a function of two independent variables, while the integral involving the unknown function is a triple integral.

The problem can be made determinate, however, by assuming that f is of a form which transforms the triple integral in (10) into a double integral. For instance, we might take

$$f(x, y, z) = f_1(x, y, z)\psi(x, y), \qquad (11)$$

where f_1 is an *assigned* function. Then substituting in (10) and performing the integration with respect to z', we obtain an integral equation of the form

$$\phi(x, y) = \int_{-\infty}^{\infty}\int (A\partial\psi/\partial x' + B\partial\psi/\partial y' + C\psi)dx'dy', \qquad (12)$$

where A, B, C are known functions of x, y, x', y', and $\psi(x', y')$ is unknown. Integrating (12) by parts, and assuming that f_1 in (11) is such as to make A, B vanish at the limits, we obtain

$$\phi(x, y) = \int_{-\infty}^{\infty}\int (-\partial A/\partial x' - \partial B/\partial y' + C)\psi(x', y')dx'dy'. \qquad (13)$$

(13) is an integral equation of standard, though singular, type, and we may assume it possesses a unique solution for ψ if the proper conditions are satisfied by the functions ϕ, A, B, C. Thus, a knowledge of V_s, I, σ_0 and the function f_1 in (11) suffices to determine σ uniquely. The function f_1 cannot be chosen completely arbitrarily on account of the convergence conditions

necessary, but it is evident that a considerable range of choice is allowed. Other choices than (11) for the functional form of f could of course be made.

The above applies to the general case where the problem possesses no special symmetry characteristics. An important case of symmetry is where σ is assumed to be function of z only. This choice reduces the triple integral in (10) to a simple integral, but on the other hand obviously requires that V_s shall be a function of r_0 (the distance from the electrode) only, so that (10) reduces to an integral equation of the form

$$\phi(r_0) = \int_{-\infty}^{\infty} K(r_0, z')[df(z')/dz']dz',$$

where K is known, and thus possesses a unique solution for f when σ_0 is known. Hence, if V_s is symmetrical with respect to the electrode and σ is assumed in advance to be a function of depth only, σ is determined uniquely from a knowledge of V_s, I, σ_0. This is the conclusion also reached by Slichter and Langer by their more exact method.

The general problem can also be made determinate by supposing that the electrode is movable, and that V_s is known at all points and for all positions of the electrode. Keeping the origin fixed and denoting the coordinates of the electrode by (ξ, η), the function ϕ in (10) now becomes a function of the four independent variables x, y, ξ, η, so that there is *more* than enough information to solve the problem. If, however, we suppose the electrode is movable along some *curve* in the surface, say $\eta = \eta(\xi)$, (10) assumes the form

$$\phi(x, y, \xi) = \int \nabla' f(\mathbf{r}') \cdot \mathbf{K}(x, y, \xi; \mathbf{r}')d\tau', \quad (14)$$

which is analogous to (12) and will possess a unique solution for f.[6] Thus, in the general case, *σ is uniquely determined from a knowledge of I, σ_0, and the surface potential for all positions of the electrode on any curve[6] at the surface.* The simplest and most natural curve to take would be a straight line, say the x-axis. The explicit expression for (14) is then

$$(1/C)V_s(x, y) - [(x-\xi)^2+y^2]^{-\frac{1}{2}} = \int \nabla' f(\mathbf{r}') \cdot \mathbf{K} d\tau', \quad (15)$$

where

$$\mathbf{K} = -(\mathbf{r}'/4\pi)[(x'-\xi)^2+y'^2+z'^2]^{-\frac{1}{2}}[(x'-x)^2+(y'-y)^2+z'^2]^{-\frac{1}{2}}$$

$(\xi, 0)$ being the coordinates of the electrode referred to the fixed origin.

Assuming that the problem has been made determinate in either of the ways mentioned above, the *complete* integral Eq. (9) may itself be solved (at least formally) by the method of successive approximations. With the assumption (11), for instance, (9) is of the form

$$\phi(x, y) = \int L\psi(x', y')dx'dy' + \int [M\psi(x', y')][N\psi(x'', y'')]dx'dy'dx''dy'' + \cdots,$$

where L, M, N, $\cdots$ are linear differential operators on the unknown function ψ, L being a function of x, y, x', y', M and N functions of x, y, x', y', x'', y'', etc. The first approximation, ψ_1, is given by

$$\phi(x, y) = \int L\psi_1 dx'dy'$$

the second approximation, ψ_2, by

$$\phi(x, y) = \int L\psi_2 dx'dy' + \int (M\psi_1)(N\psi_1)dx'dy'dx''dy''$$

[6] Certain restrictions on the curve would be necessary. For instance, it is probably necessary that the range of variation of ξ along the curve should be from $-\infty$ to ∞, since this is so for the other variables concerned.

and so on. These successive integral equations are all of the form (12) which has been discussed already. Similarly for the complete integral equation corresponding to (14).

The above considerations depend on the possibility of applying the method of successive approximations and on the existence of solutions of the integral equations, but the general conclusions as to the possibility or otherwise of determining conductivity from surface potential can hardly depend on this and may therefore be assumed to hold generally. The possibility of discontinuous solutions for σ does not appear to be allowed for, however (neither is it in the Slichter-Langer method), although such solutions may undoubtedly exist. It is conceivable, though it does not seem at all probable, that in certain cases where the above indicates a unique solution, there may be two solutions, one continuous and the other discontinuous.

4. The First Approximation when σ is a Function of Depth Only

In this section, we confine ourselves to the first approximation and to the case where $\sigma=\sigma(z)$. In this case, (9) becomes, writing r in place of r_0 (distance from electrode)

$$V_s(r)=\frac{C}{r}-\frac{C}{4\pi}\int\frac{z'df(z')/dz'}{r'^3|\mathbf{r}-\mathbf{r}'|}d\tau'. \qquad (16)$$

Performing the integration with respect to x', y',[7] we find

$$V_s(r)=\frac{C}{r}-C\int_0^\infty\frac{f'(z)}{(r^2+4z^2)^{\frac{1}{2}}}dz \qquad (17)$$

or

$$\phi(r)\equiv\frac{1}{C}V_s(r)-\frac{1}{r}=\int_0^\infty\frac{\chi(z)}{(r^2+z^2)^{\frac{1}{2}}}dz, \qquad (18)$$

where

$$\chi(z)=-\tfrac{1}{2}f'(z/2)=-(1/2\sigma)\sigma'(z/2). \qquad (19)$$

(17), or (18), is the integral equation to determine σ in this case. It is not of a known type, but it can be solved approximately by the following method which brings out the analogy with the Slichter-Langer method.

[7] See note at end of paper.

Since

$$(r^2+z^2)^{-\frac{1}{2}}=\int_0^\infty e^{-\lambda z}J_0(\lambda r)d\lambda$$

we have, substituting in (18), since it is evidently legitimate to invert the order of integration,

$$\phi(r)=\int_0^\infty\psi(\lambda)J_0(\lambda r)d\lambda, \qquad (20)$$

where

$$\psi(\lambda)=\int_0^\infty e^{-\lambda z}\chi(z)dz. \qquad (21)$$

(20) can be inverted by a theorem due to Hankel,[8] yielding

$$\psi(\lambda)=\lambda\int_0^\infty r\phi(r)J_0(\lambda r)dr, \qquad (22)$$

so that the problem is reduced to solving the integral Eq. (21).[9] This equation occurs in the theory of Heaviside's operational method, and the solution, under suitable restrictions, is[10]

$$\chi(z)=(1/2\pi i)\int_{c-i\infty}^{c+i\infty}\psi(\lambda)e^{\lambda z}d\lambda, \qquad (23)$$

where c is real and positive. This is of no use in the present case, however, for it necessarily involves a knowledge of $\psi(\lambda)$ for complex λ, and (22) only defines $\psi(\lambda)$ for real λ. We may obtain an approximate solution of (21) as follows: let us assume $\chi(z)$ can be expanded in a power series

$$\chi(z)=\sum_{n=0}^{\infty}c_nz^n;$$

substituting in (21), we find

$$\psi(\lambda)=\sum_{n=0}^{\infty}(n!c_n/\lambda^{n+1}).$$

[8] This has been pointed out by both King (reference 1) and Slichter (reference 3). King first differentiates (20) with respect to r in order to obtain a result in terms of the surface potential *gradient* dV_s/dr rather than V_s itself, since the former quantity is the more easily measured experimentally.

[9] We would be led more directly to this by solving the original Eq. (1), using cylindrical coordinates in the usual manner.

[10] See, for instance, H. W. March, Bull. Am. Math. Soc. **33**, 311 (1927).

Thus if, for $\lambda \neq 0$, $\psi(\lambda)$ can be expanded in the form

$$\psi(\lambda) = \sum_{n=1}^{\infty} (a_n/\lambda^n), \tag{24}$$

the solution of (21) can be taken to be

$$\chi(z) = \sum_{n=0}^{\infty} (a_{n+1}/n!)z^n. \tag{25}$$

This may now be compared with the Slichter-Langer method. The latter also uses a function which is equivalent to our $\psi(\lambda)$ of Eq. (20), except that it is defined with reference to the complete surface potential $V_s(r)$ rather than the first approximation $\phi(r)$. The adjustment is easily made, however, and, in the present notation, the Slichter-Langer can be concisely stated thus:

Expand $\psi(\lambda)$ in the form (24); then the solution for σ is given by

$$f'(z) = -2 \sum_{n=0}^{\infty} b_n z^n, \tag{26}$$

where $b_0 = a_1$, $b_2 = 2a_2 - a_1^2$, and in general, for $n \geq 2$, b_n is given by the recurrence formulae[11]

$$b_n = (1/n) \sum_{i=1}^{n} c_i \, \partial b_{n-1}/\partial a_i,$$

$$c_n = 2a_{n+1} + \sum_{i=1}^{n-2} a_{i+1} a_{n-i} \quad (c_1 = b_1).$$

By bearing in mind that $f'(z) = -2\chi(2z)$, (26) may be compared with our (25).

The Slichter-Langer method is *exact* if it be assumed that σ is analytic for all z and does not vanish. Under these circumstances, the function $\psi(\lambda)$ possesses an asymptotic expansion of the form (24). In many cases of practical interest, however, σ will certainly not satisfy the condition of being analytic—in particular, the case of the "layer-earth," where σ has discontinuities, is excluded. It might be urged that actual discontinuities do not occur, only steep gradients, and that an approximation should be obtained whatever the behavior of σ. A numerical example considered below will indicate that this is definitely not so; but in any case, a power series expansion is not well adapted to the case of steep gradients, and, as pointed out already, it is precisely the detection of regions where there are rapid changes in σ that is of importance in geophysical prospecting. Thus neither the Slichter-Langer nor the present approximate method is a particularly good one from the practical standpoint.

In practice, one would apply either of these methods as follows: from the observed measurements of $\phi(r) = (1/C)V_s(r) - 1/r$ the function $\psi(\lambda)$ can be calculated from (22) by numerical integration for various values of λ. Then, by plotting $\psi(1/\lambda)$ as a function of λ, some formula of interpolation can be used to express $\psi(1/\lambda)$ as a polynomial in λ which is valid approximately over a range of λ not too large; this then yields a certain number of terms of the expansion (24) valid for sufficiently great values of λ (this can be done in a formal manner even though the expansion (24) is not valid). From (25) or (26) a certain number of terms of the expansion for $f'(z)$ can then be found, from which of course σ can be found, since, from (19)

$$\sigma(z) = \sigma_0 \exp\left(\int_0^z f'(z)dz\right). \tag{27}$$

The fact that only a finite number of terms in the expansions can be found corresponds to the fact that experimentally a knowledge of $V_s(r)$ or $\phi(r)$ is confined to values of r not too great; for this means that $\psi(\lambda)$ from (22) is only known accurately for values of λ not too small, and this in turn means that only a finite number of terms in the expansion (24) can be found.

---

[11] Given implicitly by Langer (reference 4). As the calculation of the b's is rather laborious, it may be of interest for future reference to give them as far as calculated. The higher b's contain a great many terms, but the expressions simplify considerably if $a_1 = 0$ (i.e., $\sigma'(0) = 0$). As this condition would probably be satisfied nearly enough in most practical cases, we give the coefficients for this case. As far as b_5 they are:

$b_2 = 2a_3$, $b_3 = (2/3)(2a_4 - a_2^2)$, $b_4 = (2/3)(a_5 - a_2 a_3)$,

$b_5 = (2/15)(2a_6 + 6a_2^3 - a_3^2 - 2a_2 a_4)$,

$b_6 = (2/45)(2a_7 + 37a_2^2 a_3 - 2a_2 a_5 - 2a_3 a_4)$,

$b_7 = (2/315)(4a_8 - 2a_4^2 - 4a_3 a_5 + 132a_2^2 a_4 - 4a_2 a_6 + 184a_2 a_3^2 - 65a_2^4)$,

$b_8 = (1/315)(2a_9 - 2a_4 a_5 - 2a_2 a_7 - 2a_3 a_6 + 91a_3^3 - 296a_2^3 a_3 + 107a_2^2 a_5 + 382a_2 a_3 a_4)$.

Furthermore, the determination of σ can obviously only be carried out to a certain depth in practice, and this corresponds to the fact that we are limited to a finite number of terms in (25) or (26).

It was thought worth while to test how nearly the Slichter-Langer method and the present approximate method would yield correct results, even though they were not strictly applicable, when applied to some simple practical case where the behavior of σ was known in advance. For this purpose, one of the cases considered by Muskat[2] was selected, namely that of a 3-layer earth where two layers of equal depth are superimposed on an infinite layer, the conductivities in the top and bottom layers being both equal to five times that in the middle layer. This case is of such a simple and practical character, that any method which claims to practical utility should be able to deal with it. In this case, we need not actually compute the function $\psi(\lambda)$ by numerical integration from the surface potential, since the latter is itself calculated from an integral of the form (20), so that we actually have the analytical expression for $\psi(\lambda)$. In fact, with the present notation and using Muskat's results, it is easily seen that

$$\psi(\lambda)=12(e^{-\lambda}-e^{-2\lambda})/(9-10e^{-\lambda}+6e^{-2\lambda}), \quad (28)$$

where the unit of length is taken to be twice the depth of the top layer.

The Slichter-Langer method and the approximate method both fail here since the right-hand side of (28) possesses no expansion of the form (24)[12]; we may, however, apply the methods formally as explained above. For this purpose, $\psi(1/\lambda)$ was plotted against λ, and an approximate representation as a polynomial of the twelfth degree obtained from the values of the function for 13 selected values of the argument from $\lambda=0$ to $\lambda=6.0$, by using the method of interpolation for divided differences with repeated arguments.[13] This gave a fair representation of the curve for this interval, but one which fails of course in the neighborhood of $\lambda=0$. The approximate values of the first 12 of the coefficients in (24) were thus obtained, from which the coefficients in the expansion for $f'(z)$ were found as above. The result, using the approximate method, is shown in Fig. 1. The correct curve is,

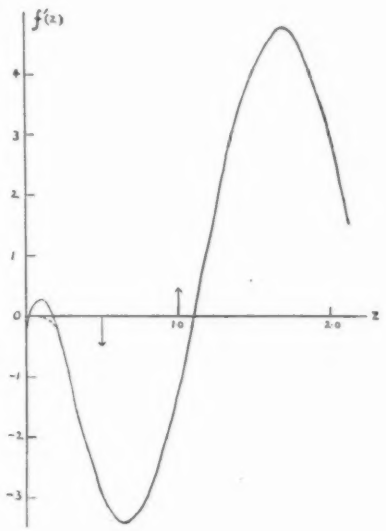


FIG. 1. Curve for $f'(z)=\sigma'(z)/\sigma(z)$ derived by the approximate method here given for the case where the conductivity is really that of a 3-layer earth, the two top layers being of equal depth and the conductivity in the top and bottom layers being five times that in the middle layer. The true curve is zero everywhere except at $z=0.5$, 1.0 where it becomes $-\infty$, $+\infty$, respectively. Unit of length = twice depth of top layer.

of course, zero everywhere except at $z=0.5$, 1.0 where it becomes $-\infty$, $+\infty$, respectively. The curve cannot safely be continued beyond about $z=2.0$, for at this value some of the terms in the expansion are large, so that any uncertainty in the coefficients is magnified; and further the highest powers of z are beginning to be effective, which would in any case render it necessary to calculate more terms. No great accuracy is claimed, but the accuracy is probably greater than would be attainable in practice.

The Slichter-Langer method was then tried, and it was fully expected that a better result would be obtained. It was found, however, that the terms in the series (26), compared with those in (25), became embarrassingly large, rendering it impossible to calculate $f'(z)$ for values of z

[12] The approximate method fails whether we adopt the method given here for the solution of the integral Eq. (18) or not; for (20) and (21) are fully equivalent to (18), and (21) shows that $\psi(\lambda)$ behaves like $1/\lambda$ (or possibly a higher power of $1/\lambda$) for large λ, which is inconsistent with (28).

[13] Whittaker and Robinson, *The Calculus of Observations*, p. 27.

greater than about 0.25 without making the form of the curve depend appreciably on the number of terms retained. In fact, the coefficients as far as calculated (b_8) have every appearance of giving a series which is divergent for all z. This is because the coefficients b_n in (26) are polynomial functions of the a's with a number of terms which increases with n, and in the present case the signs of the a's are such that these terms always reinforce each other, thus making the b_n increase rapidly with n. Neither can the series be used as an asymptotic series. It thus appears that the Slichter-Langer method cannot be used, even approximately, for the case under consideration.

Considering the curve for $f'(z)$ derived from the approximate method as given by Fig. 1, the agreement between the true and calculated curves is certainly not good, but it may be said that the calculated curve does at least afford some indication of the true state of affairs. The small variations near the origin have evidently only been introduced through the inaccuracy in the representation of $\psi(1/\lambda)$ as a polynomial in λ near $\lambda=0$; the real curve for $\psi(1/\lambda)$ is very flat near the origin, and a comparison of (24) and (25) shows that the same should then be true for the curve $\chi(z)$, and hence $f'(z)$. It is therefore permissible to "flatten out" the curve for $f'(z)$ as shown by the dotted part of it near the origin. The fairly sharp and large negative minimum[14] then indicates a region of fairly rapid decrease in σ, while the positive maximum indicates a region of fairly rapid increase in σ, thus giving some indication of the two layer-boundaries.

The curve for σ/σ_0 derived from (27) by numerical integration is shown in curve I of Fig. 2. The mean depth of the first transition region indicated is about right, but that of the second transition region is too great by about sixty percent. The increase in σ at the second

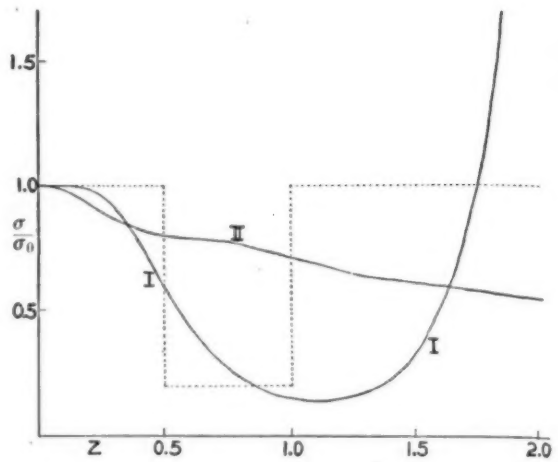


FIG. 2. Curve for σ/σ_0 for the 3-layer earth considered in Fig. 1. Curve I is derived by the approximate method, curve II by the "surface gradient characteristic" method (Eq. (29)). The dotted curve is the true one. The unit of length is the same as for Fig. 1.

transition region is much too great, showing that the solution is beginning to fail badly after about $z=1.8$. A quantitative agreement between calculated and true values of σ can hardly be hoped for, however; the most that can be expected is a qualitative agreement between the curves for $f'(z)$ or σ/σ_0 provided z is not too large. The indicated minimum value of σ/σ_0 agrees remarkably well with the true value (1/5), but this agreement is probably fortuitous.

It is of interest to compare the curve for σ/σ_0 obtained from a formula, derived from very crude arguments, which is given in Eve and Keyes' book on geophysical prospecting,[15] which amounts, in the present notation to taking

$$\sigma_0/\sigma(z) = -(z^2/C)(dV_s(z)/dz), \qquad (29)$$

C being defined as before ($I/2\pi\sigma_0$). The expression on the right of (29) is commonly referred to as the "apparent resistivity," but has been more aptly termed by King[1] the "surface gradient characteristic." The right-hand side of (29) was roughly estimated from Muskat's curve for $(r/C)V_s(r)$ for this case, and the resulting curve for σ/σ_0 shown in curve II of Fig. 2. The curve decreases very gradually to a flat minimum at about $z=3.5$, thereafter increasing slowly to its original value, and thus affords no indication

[14] The magnitude of $f'(z)$ and the sharpness of its extrema depend entirely on the unit of length adopted, since $f'(z)$ is of dimensions (length)$^{-1}$. In practical cases, a unit of length would be adopted in conformity with the extent of the surface potential measurements and the sharpness of the extrema judged from geophysical considerations. In the present case, the origin of the problem provides a natural unit length.

[15] Eve and Keyes, *Applied Geophysics*, 110.

whatever of the two transition regions. It seems rather surprising that a method as inexact as this should have had any success in practical field work.

5. Concluding Remarks

The object of the present paper has been firstly to give a general theory—even if only a formal one in certain respects—of the calculation of potentials when σ is known and of the converse problem of determining σ from a knowledge of the surface potential, and secondly to examine the practicability of applying the "first approximation" to solve the latter problem in the case where σ is a function of depth only. The above example indicates that the method might, with caution, be applied to give an indication of conditions even when the more exact Slichter-Langer method fails. This is because the series (25) is likely to be convergent when obtained as explained above even when the corresponding series (26) is divergent. Admittedly the example taken is unfair to the Slichter-Langer method, but it does indicate that the method may fail to give even an approximately correct answer in cases of practical importance. Where applicable, the Slichter-Langer method is undoubtedly superior to the present one.

It is suggested that in practical cases both methods might be used (most of the computation is identical for the two) and if the Slichter-Langer method obviously fails, as it does in the above example, the present method might be used. A serious disadvantage to both methods from the practical point of view is the large amount of computation required for each individual case. If a more rapid method than the one given here could be devised for finding an approximate solution of the integral Eq. (18), this would be an appreciable advantage.

As regards the more general problem where σ is not a function of depth only, a possibility is opened up by the integral Eq. (15) which, if it could be solved, would give, in first approximation, the conductivity when the surface potential is known for all positions of the electrode on a straight line. But the equation seems to hold little possibility of even an approximate analytical solution, and any solution by numerical methods, seeing that we have to deal with a triple integral, would probably involve a prohibitive amount of labor. What is required from the standpoint of geophysical prospecting is a method which will give, without too much computation, the approximate behavior of σ, and which will, in particular, indicate the occurrence of regions of rapid change of σ. It cannot be said that such a method has yet been given, even for the relatively simple case where σ is a function of depth only. Until such a method has been devised, the cumbrous one of "trial and error" must remain of importance.

Note. The Evaluation of an Integral

The integration with respect to x', y' in (16) which we have to perform is

$$J(r, z') = \int_{-\infty}^{\infty}\int \frac{dx'dy'}{r'^3|\mathfrak{r}-\mathfrak{r}'|}. \qquad \text{I}$$

A direct evaluation seems to be rather difficult, but it can be found indirectly as follows: taking polar coordinates with the polar axis vertically *upwards*, J is seen to be the potential at $(r, \pi/2)$ of a surface distribution of density $1/r'^3$ (r' being the distance from the origin) in a horizontal plane at depth z'. Considering the value of such a potential at the point (r, θ), it can be expanded in the form

$$P(r, \theta) = \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}}\right)P_n(\cos\theta), \qquad \text{II}$$

where the A's and B's are functions of z' only. To find the coefficients, put $\theta = 0$. The potential can then be found easily and yields the value $2\pi/z'(r+2z')$. Expanding this in powers of r and comparing with II, we find

$$P(r, \theta) = \frac{2\pi}{z'r}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2z'}{r}\right)^n P_n(\cos\theta), \qquad (r > 2z')$$

$$= \frac{2\pi}{z'^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{r}{2z'}\right)^n P_n(\cos\theta), \qquad (r < 2z'). \quad \text{III}$$

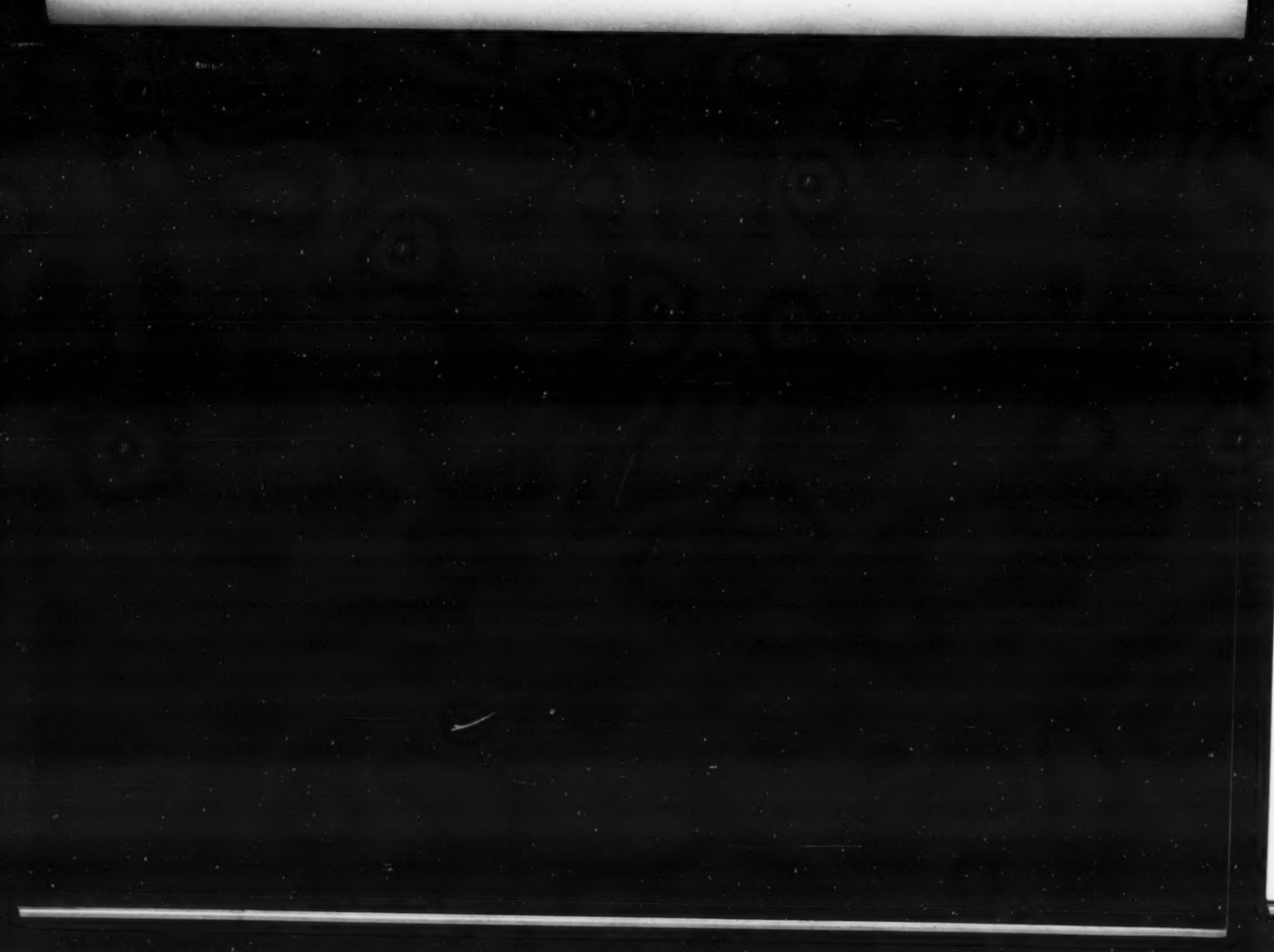
Now from the well-known formula

$$P_n(x) = (1/2^n n!)(d/dx)^n(x^2-1)^n$$

it can easily be shown that $P_{2n+1}(0)=0$, while $P_{2n}(0)$ is equal to the coefficient of x^n in the expansion of $(1+x)^{-1/2}$. Hence from III we find, putting $\theta=\pi/2$,

$$J = P(r, \pi/2) = 2\pi/z'(r^2+4z'^2)^{\frac{1}{2}}.$$

Substituting this value of J in the integral of (16) gives (17) of the text.



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